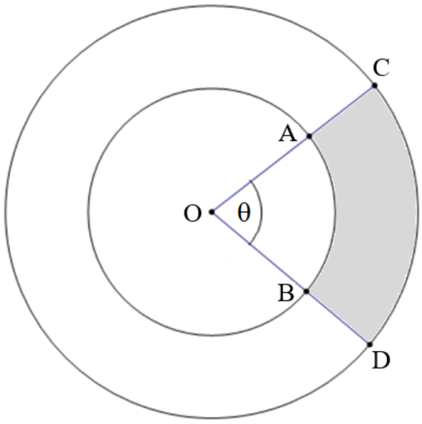
Mathematics: analysis and approaches
Higher Level
Paper 2 (set C)

worked solutions

1. [Maximum mark: 5]



The diagram below shows two circles which have the same centre O. The smaller circle has a radius of 12 cm and the larger circle has a radius of 20 cm. The two arcs AB and CD have the same central angle θ, where $\theta = 1.3$ radians.
Find the area of the shaded region.

Solution:

area of shaded region = (area of sector COD) – (area of sector AOB)

area of sector COD $= \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 20^2 \cdot 1.3 = 260 \text{ cm}^2$

area of sector AOB $= \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 12^2 \cdot 1.3 = 93.6 \text{ cm}^2$

thus, area of shaded region $= 260 - 93.6 = 166.4 \approx 166 \text{ cm}^2$

2. [Maximum mark: 5]

Two lines have the vector equations $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

Find the obtuse angle between the lines.

Solution:

Let θ be the angle between the lines.

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}\right|\left|\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}\right|} = \frac{7}{\sqrt{6}\sqrt{14}} \Rightarrow \theta = \cos^{-1}\left(\frac{7}{\sqrt{6}\sqrt{14}}\right) \approx 40.203...^\circ$$

obtuse angle between the lines $\approx 180^\circ - 40.203...^\circ \approx 139.797...^\circ \Rightarrow \theta \approx 140^\circ$

3. [Maximum mark: 5]

Find the coefficient of the x^3 term in the expansion of $\left(\frac{2}{3}x+3\right)^8$.

Solution:

general term of expansion: $\binom{8}{r}\left(\frac{2}{3}x\right)^{8-r}3^r$

considering exponent of x: $8-r=3 \Rightarrow r=5$

thus, coefficient of x^3 term $=\binom{8}{5}\left(\frac{2}{3}\right)^3 3^5 = 56\cdot\frac{8}{27}\cdot 243 = 4032$

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4. [Maximum mark: 6]

The table below shows the marks earned on a quiz by a group of students.

Mark	1	2	3	4	5
Number of students	8	7	c	9	1

The median is 3 and the mode is 4 for the set of marks. Find the **three** possible values of c.

Solution:

Since 4 is the mode, then $c<9$

The total number of students is $8+7+c+9+1=c+25$

Since 3 is the median, then $\frac{c+25}{2}>8+7 \Rightarrow c+25>30 \Rightarrow c>5$

Thus, the three possible values of c are 6, 7, 8

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5. [Maximum mark: 6]

Consider the complex number $z=\frac{\sqrt{2}}{1-\mathrm{i}}-\mathrm{i}$.

(a) Show that z can be expressed, in the form $x+y\mathrm{i}$, as $z=\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{2}-2}{2}\right)\mathrm{i}$. [2]

(b) (i) Find the **exact** value of the modulus of z.

(ii) Find the argument θ of z, where $-\pi<\theta\le\pi$. [4]

Solution:

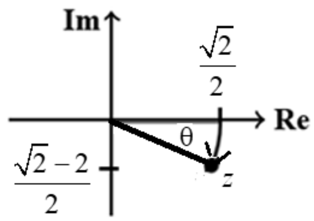
(a) $z=\frac{\sqrt{2}}{1-\mathrm{i}}\cdot\frac{1+\mathrm{i}}{1+\mathrm{i}}-\mathrm{i}=\frac{\sqrt{2}+\mathrm{i}\sqrt{2}}{1-\mathrm{i}^2}-\mathrm{i}=\frac{\sqrt{2}+\mathrm{i}\sqrt{2}}{2}-\mathrm{i}=\frac{\sqrt{2}+\mathrm{i}\sqrt{2}-2\mathrm{i}}{2}$ Thus, $z=\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{2}-2}{2}\right)\mathrm{i}$

solution continued on next page >>

Q.5 solution continued

(b) modulus: $|z|=\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}-2}{2}\right)^2}=\sqrt{\frac{1}{2}+\frac{2-4\sqrt{2}+4}{4}}=\sqrt{\frac{2}{4}+\frac{6-4\sqrt{2}}{4}}=\sqrt{\frac{8-4\sqrt{2}}{4}}=\sqrt{2-\sqrt{2}}$

argument: $\arg(z)=\theta=\tan^{-1}\left(\frac{\frac{\sqrt{2}-2}{2}}{\frac{\sqrt{2}}{2}}\right)=\tan^{-1}\left(\frac{\sqrt{2}-2}{\sqrt{2}}\right)=-0.392699...$



thus, $\theta\approx-0.393$

6. [Maximum mark: 7]

(a) Express $\frac{1}{2x^2+7x-4}$ in partial fractions; i.e. as the sum of two fractions. [4]

(b) Given that $\int_1^4 \frac{9}{2x^2+7x-4}\,dx=\ln k$, find the **exact** value of k. [3]

Solution:

(a) $\frac{1}{2x^2+7x-4}=\frac{1}{(2x-1)(x+4)}=\frac{A}{2x-1}+\frac{B}{x+4}$

multiply both sides by $(2x-1)(x+4)$, to give $1=A(x+4)+B(2x-1)$

let $x=\frac{1}{2}$: $1=A\left(\frac{1}{2}+4\right)\Rightarrow\frac{9}{2}A=1\Rightarrow A=\frac{2}{9}$

let $x=-4$: $1=B(-8-1)\Rightarrow -9B=1\Rightarrow B=-\frac{1}{9}$

thus, $\frac{1}{2x^2+7x-4}=\frac{1}{(2x-1)(x+4)}=\frac{\frac{2}{9}}{2x-1}-\frac{\frac{1}{9}}{x+4}$ or $=\frac{2}{9(2x-1)}-\frac{1}{9(x+4)}$

(b) $\int_1^4\frac{9}{2x^2+7x-4}\,dx=9\int_1^4\frac{1}{2x^2+7x-4}\,dx=9\int_1^4\left(\frac{2}{9(2x-1)}-\frac{1}{9(x+4)}\right)\mathrm{d}x=\int_1^4\left(\frac{2}{2x-1}-\frac{1}{x+4}\right)\mathrm{d}x$

$=\left[\ln|2x-1|-\ln|x+4|\right]_1^4=(\ln 7-\ln 8)-(\ln 1-\ln 5)=\ln\frac{7}{8}-0+\ln 5=\ln\frac{35}{8}$ thus, $k=\frac{35}{8}$

7. [Maximum mark: 6]

(a) Write down the Maclaurin expansion of e^x up to the term in x^4. [1]

(b) Find the Maclaurin expansion of e^{x^2} up to the term in x^4. [2]

(c) Hence, find the Maclaurin expansion of e^{x+x^2} up to the term in x^4. [3]

Solution:

(a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ or $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

(b) $e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{6} + \frac{(x^2)^4}{24} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \frac{x^8}{24}$

(c) $e^{x+x^2} = e^x \cdot e^{x^2} = \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \frac{x^8}{24}\right)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}\right)$

$= \left(1 + x^2 + \frac{x^4}{2}\right)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}\right) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + x^2 + x^3 + \frac{x^4}{2} + \ldots + \frac{x^4}{2} + \ldots$

$= 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3 + \frac{25}{24}x^4$

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8. [Maximum mark: 6]

Consider the following system of equations

$$\begin{aligned} 2x + y + 6z &= 0 \\ 4x + 3y + 14z &= 4 \\ 2x - 2y + (\alpha - 2)z &= \beta - 12 \end{aligned}$$

Find the conditions on α and β for which

(a) the system has no solutions; [2]

(b) the system has only one solution; [2]

(c) the system has an infinite number of solutions. [2]

Solution: using row operations

$$\left(\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 4 & 3 & 14 & 4 \\ 2 & -2 & \alpha-2 & \beta-12 \end{array}\right) \xrightarrow{(R1-R3\to R3)} \left(\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 4 & 3 & 14 & 4 \\ 0 & -3 & \alpha-8 & \beta-12 \end{array}\right)$$

$$\xrightarrow{(R2-2R1\to R2)} \left(\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & -3 & \alpha-8 & \beta-12 \end{array}\right)$$

$$\xrightarrow{(R3-3R2\to R3)} \left(\begin{array}{ccc|c} 2 & 1 & 6 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta \end{array}\right)$$

Thus,

(a) no solutions when $\alpha = 2$ and $\beta \neq 0$

(b) one solution when $\alpha \neq 2$

(c) infinite solutions when $\alpha = 2$ and $\beta = 0$

9. [Maximum mark: 7]

Consider the differential equation $x\frac{dy}{dx}+3y=\frac{1}{x}$, $x>0$ such that $y=1$ when $x=1$. Show that the solution to this differential equation is $y=\frac{x^2+1}{2x^3}$.

Solution:

$$x\frac{dy}{dx}+3y=\frac{1}{x} \Rightarrow \frac{dy}{dx}+\frac{3}{x}y=\frac{1}{x^2} \qquad \text{integrating factor, IF}=e^{\int\frac{3}{x}dx}=e^{3\ln x}=x^3$$

$$x^3\left(\frac{dy}{dx}+\frac{3}{x}y\right)=x^3\left(\frac{1}{x^2}\right) \Rightarrow \int x^3\left(\frac{dy}{dx}+\frac{3}{x}y\right)dx=\int x\,dx \Rightarrow x^3y=\frac{1}{2}x^2+C$$

$$x=1 \text{ when } y=1:\ (1)^3(1)=\frac{1}{2}(1)^2+C \Rightarrow C=\frac{1}{2}$$

$$x^3y=\frac{1}{2}x^2+\frac{1}{2}=\frac{x^2+1}{2} \Rightarrow y=\frac{x^2+1}{2x^3} \qquad \textit{QED}$$

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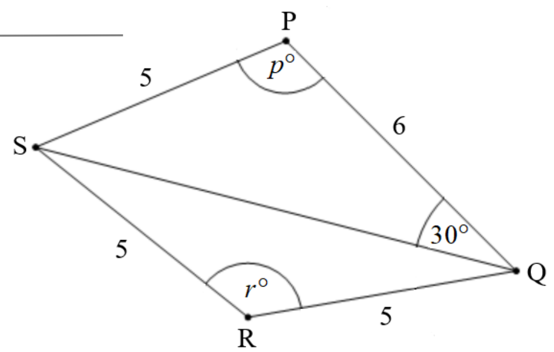
10. [Maximum mark: 15]



The diagram shows the quadrilateral PQRS. Angle QPS and angle QRS are obtuse.

$PQ=6\,\text{cm}$, $QR=5\,\text{cm}$, $RS=5\,\text{cm}$, $PS=5\,\text{cm}$,
$P\hat{Q}S=30°$, $Q\hat{P}S=p°$, $Q\hat{R}S=r°$

(a) Use the sine rule to show that $QS=10\sin p$. [1]

(b) Use the cosine rule in triangle PQS to find another expression for QS. [3]

(c) (i) Hence, find p, giving your answer to two decimal places.

(ii) Find QS. [6]

(d) (i) Find r.

(ii) Hence, or otherwise, find the area of triangle QRS. [5]

Solution:

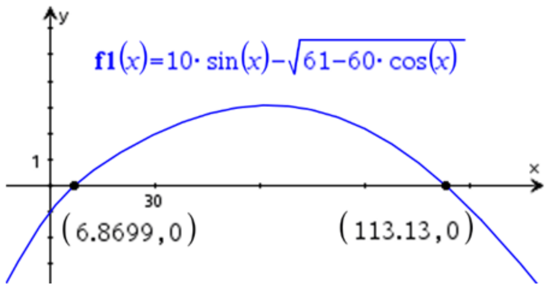
(a) $\frac{\sin 30°}{5}=\frac{\sin p}{QS} \Rightarrow QS=\frac{5}{1/2}\sin p \Rightarrow QS=10\sin p$ ***QED***

(b) $QS^2=5^2+6^2-2(5)(6)\cos p \Rightarrow QS=\sqrt{61-60\cos p}$

(c) (i) solve the equation: $10\sin p=\sqrt{61-60\cos p}$

clearly, $90<p<180$; solve with graph or GDC solver

nSolve$(10\cdot\sin(p)=\sqrt{61-60\cdot\cos(p)},p)|p>6.87$
113.130102354



thus, $p\approx 113.13$

solution continued on next page >>

Q.10 solution continued

(c) (ii) $QS=\sqrt{61-60\cos(113.130102354°)}\approx 9.19615... \Rightarrow QS\approx 9.20\text{ cm}$

(d) (i) using cosine rule in triangle QRS

$$(9.19615...)^2=5^2+5^2-2(5)(5)\cos r \Rightarrow r=\cos^{-1}\left(\frac{50-(9.19615...)^2}{50}\right)\approx 133.7398... \Rightarrow r\approx 134$$

(ii) area of triangle $QRS=\frac{1}{2}(5)(5)\sin(133.7398...°)\approx 9.0310889...\approx 9.03\text{ cm}^2$

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11. [Maximum mark: 21]

A continuous random variable X has probability density function f defined by

$$f(x)=\begin{cases}\frac{\pi}{3}\sin\left(\frac{\pi}{2}x\right), & 0\le x\le 1\\ mx+b, & 1\le x\le k\\ 0, & \text{otherwise}\end{cases}$$

(a) Given that f is continuous on the interval $0\le x\le k$ and that the graph of f intersects the x-axis at $(k,0)$, show that $k=\frac{\pi+2}{\pi}$. [5]

(b) Find the value of m and the value of b. [3]

(c) Sketch the graph of $y=f(x)$. [2]

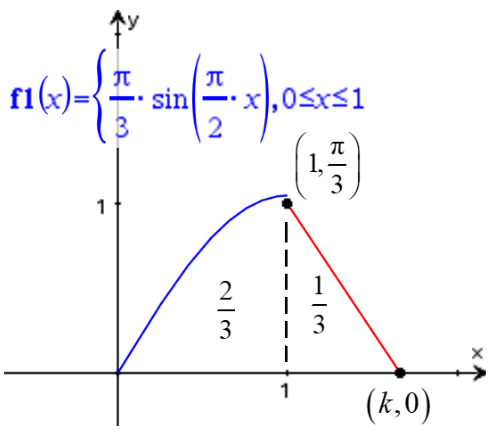
(d) Write down the mode of X. [1]

(e) Given that $\int_1^{\frac{\pi+2}{\pi}}\left[x(mx+b)\right]dx=\frac{3\pi+2}{9\pi}$, find the **exact** value of the mean of X. [7]

(f) Find the value of the median of X. [3]

<u>**Solution**</u>:

(a) $\frac{\pi}{3}\int_0^1\sin\left(\frac{\pi}{2}x\right)dx=-\frac{2}{3}\cos\left(\frac{\pi}{2}x\right)\Big]_0^1=-\frac{2}{3}(0-1)=\frac{2}{3}$



The line segment (on $y=mx+b$) must start at $\left(1,\frac{\pi}{3}\right)$

because f is a continuous function and it ends at $(k,0)$.

Since total area under p.d.f. is one, the area of

triangular region must be $1/3$. Height of triangle is $\frac{\pi}{3}$

and base is $k-1$. Hence, $\frac{1}{2}\cdot\frac{\pi}{3}(k-1)=\frac{1}{3} \Rightarrow k=\frac{\pi+2}{\pi}$ ***QED***

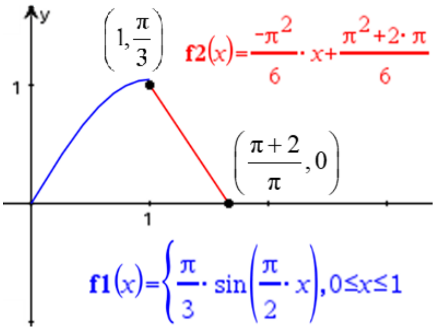
solution continued on next page >>

Q.11 solution continued

(b) Equation of line passing through $\left(1,\frac{\pi}{3}\right)$ and $\left(\frac{\pi+2}{\pi},0\right)$: $m=\dfrac{\frac{\pi}{3}-0}{1-\frac{\pi+2}{\pi}}=\dfrac{\frac{\pi}{3}}{\frac{\pi}{\pi}-\frac{\pi+2}{\pi}}=\dfrac{\frac{\pi}{3}}{-\frac{2}{\pi}}=-\dfrac{\pi^2}{6}$

$$y=-\frac{\pi^2}{6}(x-1)=-\frac{\pi^2}{6}x+\frac{\pi^2}{6}+\frac{\pi}{3}=-\frac{\pi^2}{6}x+\frac{\pi^2+2\pi}{6}$$ Thus, $m=-\dfrac{\pi^2}{6}$ and $b=\dfrac{\pi^2+2\pi}{6}$

(c)



(d) Mode is $x=1$

(e) $$\mathrm{E}(X)=\frac{\pi}{3}\int_0^1\left[x\sin\left(\frac{\pi}{2}x\right)\right]dx+\int_1^{\frac{\pi+2}{\pi}}\left[x\left(-\frac{\pi^2}{6}x+\frac{\pi^2+2\pi}{6}\right)\right]dx==\frac{\pi}{3}\int_0^1\left[x\sin\left(\frac{\pi}{2}x\right)\right]dx+\frac{3\pi+2}{9\pi}$$

Integration by parts: $$\int x\sin\left(\frac{\pi}{2}x\right)dx=-\frac{2x}{\pi}\cos\left(\frac{\pi}{2}x\right)+\frac{2}{\pi}\int\cos\left(\frac{\pi}{2}x\right)dx$$

$u=x\ \Rightarrow\ du=dx$ $$=-\frac{2x}{\pi}\cos\left(\frac{\pi}{2}x\right)+\frac{2}{\pi}\cdot\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)$$

$dv=\sin\left(\frac{\pi}{2}x\right)\ \Rightarrow\ v=-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)$ $$=-\frac{2x}{\pi}\cos\left(\frac{\pi}{2}x\right)+\frac{4}{\pi^2}\sin\left(\frac{\pi}{2}x\right)$$

$$\mathrm{E}(X)=\frac{\pi}{3}\int_0^1 x\sin\left(\frac{\pi}{2}x\right)dx+\frac{3\pi+2}{9\pi}$$
$$=\frac{\pi}{3}\left[-\frac{2x}{\pi}\cos\left(\frac{\pi}{2}x\right)+\frac{4}{\pi^2}\sin\left(\frac{\pi}{2}x\right)\right]_0^1+\frac{3\pi+2}{9\pi}$$
$$=\left[-\frac{2x}{3}\cos\left(\frac{\pi}{2}x\right)+\frac{4}{3\pi}\sin\left(\frac{\pi}{2}x\right)\right]_0^1+\frac{3\pi+2}{9\pi}$$
$$=\left(0+\frac{4}{3\pi}\right)-(0-0)+\frac{3\pi+2}{9\pi}$$
$$=\frac{4}{3\pi}+\frac{3\pi+2}{9\pi}$$ thus, $\mathrm{E}(X)=\dfrac{3\pi+14}{9\pi}$

(f) Let m be the median. Since $\frac{\pi}{3}\int_0^1\sin\left(\frac{\pi}{2}x\right)dx=\frac{2}{3}$, then $0<m<1$.

Solve for m in the equation $\frac{\pi}{3}\int_0^m\sin\left(\frac{\pi}{2}x\right)dx=\frac{1}{2}$.

$m\approx 0.839$

$\text{nSolve}\left(\frac{\pi}{3}\cdot\int_0^m\sin\left(\frac{\pi}{2}\cdot x\right)dx=\frac{1}{2},m\right)$

0.83913875349

12. [Maximum mark: 21]

The function g is defined as $g(x) = e^x + \dfrac{1}{2e^x}$, $x \in \mathbb{R}$.

(a) (i) Explain why the inverse function g^{-1} does not exist.

(ii) The line L intersects the curve $y = g(x)$ at points A and B where $x = -1$ at A and $x = 1$ at B. Show that the equation of L is $y = \dfrac{e^2 - 1}{4e}x + \dfrac{3e^2 + 3}{4e}$.

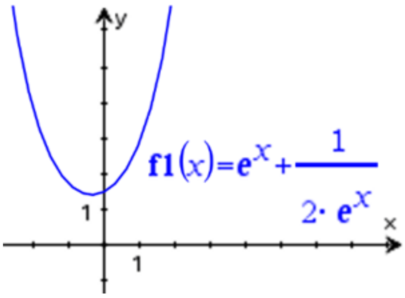
(iii) Point C is on the curve $y = g(x)$. The line tangent to the curve $y = g(x)$ at C is parallel to L. Find the coordinates of C. [13]

(b) The domain of g is now restricted to $x \geq 0$.

(i) Find an expression for $g^{-1}(x)$.

(ii) Find the volume generated when the region bounded by the curve $y = g(x)$ and the lines $x = 0$ and $y = 4$ is rotated through an angle of 2π radians about the y-axis. [8]

Solution:



(a) (i) g is not a one-to-one function since it fails the horizontal line test because a horizontal line can intersect the graph at more than one point; hence, g^{-1} does not exist

(ii) $g(1) = e + \dfrac{1}{2e} = \dfrac{2e^2}{2e} + \dfrac{1}{2e} = \dfrac{2e^2 + 1}{2e}$

$g(-1) = e^{-1} + \dfrac{1}{2e^{-1}} = \dfrac{1}{e} + \dfrac{e}{2} = \dfrac{2}{2e} + \dfrac{e^2}{2e} = \dfrac{2 + e^2}{2e}$

the coordinates A and B are $A\left(1, \dfrac{2e^2 + 1}{2e}\right)$ and $B\left(-1, \dfrac{2 + e^2}{2e}\right)$

gradient of L: $m = \dfrac{\dfrac{2e^2 + 1}{2e} - \dfrac{2 + e^2}{2e}}{1 - (-1)} = \dfrac{\dfrac{e^2 - 1}{2e}}{2} = \dfrac{e^2 - 1}{4e}$

equation of L: $y - \dfrac{2e^2 + 1}{2e} = \dfrac{e^2 - 1}{4e}(x - 1) \Rightarrow y = \dfrac{e^2 - 1}{4e}x - \dfrac{e^2 - 1}{4e} + \dfrac{2e^2 + 1}{2e}$

$y = \dfrac{e^2 - 1}{4e}x + \dfrac{-e^2 + 1}{4e} + \dfrac{4e^2 + 2}{2e} \Rightarrow y = \dfrac{e^2 - 1}{4e}x + \dfrac{3e^2 + 3}{4e}$ ***QED***

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Q.12 solution continued

(a) (iii) $g(x)=\mathrm{e}^x+\left(2\mathrm{e}^x\right)^{-1} \Rightarrow g'(x)=\mathrm{e}^x-\left(2\mathrm{e}^x\right)^{-2}\left(2\mathrm{e}^x\right)=\mathrm{e}^x-\dfrac{2\mathrm{e}^x}{\left(2\mathrm{e}^x\right)^2}=\mathrm{e}^x-\dfrac{1}{2\mathrm{e}^x}$

find the value of x such that the derivate is equal to the gradient of line L

hence, solve the equation $\mathrm{e}^x-\dfrac{1}{2\mathrm{e}^x}=\dfrac{\mathrm{e}^2-1}{4\mathrm{e}}$

$$\mathrm{nSolve}\left(e^x-\frac{1}{2\cdot e^x}=\frac{e^2-1}{4\cdot e},x\right) \quad 0.057811016577$$

solving on GDC

$x\approx 0.057811016577...$

$g(0.057811016577...)\approx 1.53142889531...$ coordinates of C are approximately $(0.0578, 1.53)$

(b) (i) $g(x)=\mathrm{e}^x+\dfrac{1}{2\mathrm{e}^x},\ x\geq 0$

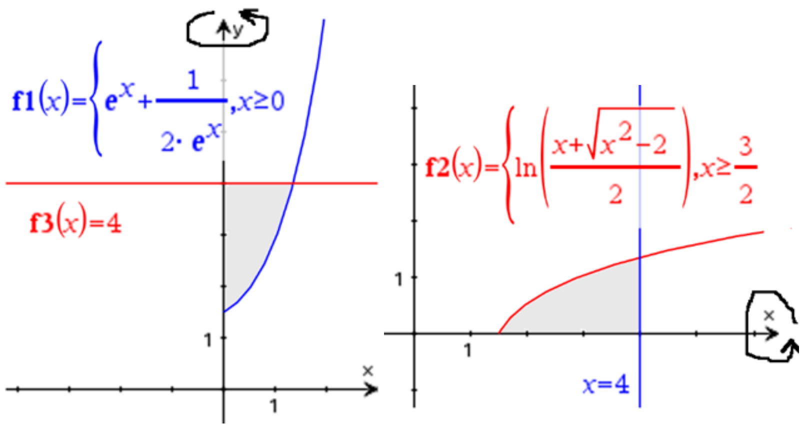
$y=\mathrm{e}^x+\dfrac{1}{2\mathrm{e}^x} \Rightarrow x=\mathrm{e}^y+\dfrac{1}{2\mathrm{e}^y} \Rightarrow 2x\mathrm{e}^y=2\left(\mathrm{e}^y\right)^2+1 \Rightarrow 2\left(\mathrm{e}^y\right)^2-2x\mathrm{e}^y+1=0$

solve for e^y: $\mathrm{e}^y=\dfrac{2x\pm\sqrt{4x^2-4(2)(1)}}{2(2)}=\dfrac{2x\pm\sqrt{4x^2-8}}{4}=\dfrac{2x\pm 2\sqrt{x^2-2}}{4}=\dfrac{x\pm\sqrt{x^2-2}}{2}$

$y=\ln\left(\dfrac{x\pm\sqrt{x^2-2}}{2}\right) \Rightarrow g^{-1}(x)=\ln\left(\dfrac{x+\sqrt{x^2-2}}{2}\right)$ need to find domain for $g^{-1}(x)$

$g(x)=\mathrm{e}^x+\dfrac{1}{2\mathrm{e}^x},\ x\geq 0 \Rightarrow g(0)=\dfrac{3}{2}$ thus, $g^{-1}(x)=\ln\left(\dfrac{x+\sqrt{x^2-2}}{2}\right)$ where $x\geq\dfrac{3}{2}$

(ii) the volume of the solid formed by rotating the region bounded by $g(x)$ about the y-axis is equal to the volume of the solid formed by rotating the region bounded by $g^{-1}(x)$ about the x-axis



volume $=\pi\displaystyle\int_{3/2}^{4}\left[\ln\left(\dfrac{x+\sqrt{x^2-2}}{2}\right)\right]^2\mathrm{d}x\approx 6.9086056902...$

$$\pi\cdot\int_{\frac{3}{2}}^{4}\left(\ln\left(\frac{x+\sqrt{x^2-2}}{2}\right)\right)^2 dx \quad 6.9086056902$$

volume ≈ 6.91 units^3